# The running-cone method for the interpretation of conical fold geometries: an example from the Badia Valley, Northern Dolomites (NE Italy) 

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#### Abstract

The running-cone method is a straightforward means of analysing the geometry of conical folds over a range of scales from outcrop to regional map. The method is based on a 3-D directional averaging process in which strike-and-dip data from natural folds may be utilized to constrain the conical geometry of regional fold sets. The orientations of local cone axes are obtained from a sequence of measurements of poles to bedding using triplets of adjacent data. The repetition of the procedure using the axes orientations yields the orientations of the poles to a surface smoother than that of the original bedding. The final steps of any single data sequence analysis, deemed significant when showing permanent orientation distributions, may highlight conical fold geometries not readily apparent from the cumulative data set. The introduction of a threshold to remove adjacent data with subparallel orientations speeds the convergence of the analysis to a unique solution. The method is tested using natural conical folds in the Dolomites, northern Italy, producing three preferred fold axes plunging to the NE, SE and WSW. These orientations are consistent with the regional fold trends.


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## 1. Introduction

Natural conical folds are typically observed where non co-axial deformational phases are superimposed (Reynolds and Holmes, 1954; Nicholson, 1959; Wilson, 1967; Ramsay and Sturt, 1973; Dubey and Cobbold, 1977; Stauffer, 1988; Stewart, 1993). Geometrical cones are surfaces on which all straight lines form the same semi-apical angle with their axis. This definition includes the cones with zero, as well as those with $\pi$, apical angles that correspond either to straight lines or to planes, and thus represent the degenerates of the conical surface into such geometrical forms (Nicol, 1993). A natural fold may be approximated by a conical surface geometry with no loss of generality. Cylindrical-fold shapes are more often documented in the literature than conical folds. As in the case for cylindrical folds, the analysis of meso-scale conical folds may provide information about the geometries of regional-scale geological folds. The straight-

[^0]forward procedure proposed here may be used in areas where conical shapes are the rule rather than the exception, and where folds are present over a range of scales.

The purpose of the analysis is the geometric description of progressively larger enveloping surfaces (Fig. 7-11 in Ramsay, 1967; Fig. 1) of conical folds. Best results are to be expected where the conical fold geometries are variable and a significant number of measurements of the folded surfaces are collected. In addition rather than forcing sets of conicalfold data into single cumulative cylindrical-fold representations, the axis orientations of subsets of the conical data (i.e. in this case subsets comprising three adjacent data points) may be found and such axes in turn used to constrain the geometry of larger conical folds possibly associated with them.

The purpose of this paper is to develop a method to utilise strike-and-dip measurements of folded surfaces to describe the geometry of conical folds between outcrop and map scales. The intermediate steps of the analysis may offer geometrical information about fold orientations and geometries over a range of scales from outcrop to regional geological map.


Fig. 1. Three-dimensional synthetic example of a conically folded surface. Its lower-order surface is also shown. The broken lines represent the higherorder conical fold axes defining the lower-order surface.

An effort has been made to show that the running-cone method, presented here, may be used successfully to identify the orientation of at least one of the dominant non co-axial superposed deformational phases where differentwavelength folds intersect producing conical shapes. The methodology proposed here is tested using field data from the lower Alpine sequence in the northern Dolomites (NE Italy).

## 2. The running-cone method

The method is a 3-D directional analysis designed to extract geometrical information on conical folds over a range of scales by plotting spatially ordered bedding planes on stereographic projections. Ultimately the analysis yields pole orientations to the enveloping surface of larger folds. Although no information is provided about fold wavelengths and/or amplitudes, the more closely spaced the data, the finer the geometrical details resolved.

Obviously the results obtained from data collected along a given section cannot be considered representative of the structures in the surroundings and elementary spectral considerations, which underlie the running-cone method, make clear that wavelengths extracted from the measurements cannot exceed the sampled length. Hence the results of the running-cone analysis may be representative of the outcrop to the extent of wavelengths shorter than that of the exposure itself.

Contrary to cylindrical folds that display axes with constant orientation, conically folded sequences produce fold axes which are generally neither parallel to each other nor lying in planes. Hence the subsequent application of the same method to the fold axes orientations, as obtained from the analysis first step, yields the pole orientations to a lowerorder surface.

Unlike vectorial data, bi-directional data are not characterized by positive or negative sides. Owing to this lack of sign in the data, the orientation of any three poles fully defines four alternative conical surfaces (Fig. 2). They are generally characterised by four different apical angles that, in the absence of a field estimate of the fold axis orientation, do not provide an unequivocal estimate of the cone axis orientation. The criterion used here to discrimi-


Fig. 2. Equal-area, lower-hemisphere projection showing the four alternative cones determined by the same three pole orientations to the folded surface. Inset depicts the polarity of the three poles according to the possible combinations (positive downwards and negative upwards). A fourth pole orientation permits the selection of the most likely cone.
nate between the four alternative cones is to select the conical surface from which a fourth additional pole shows the smallest angular divergence. The fourth pole may occasionally be angularly equidistant from two of the alternative cones; in this case the orientation of a fifth pole is taken into consideration and so on.

The methodology is outlined below.
(1) Order the bedding (i.e. the fold surface) data progressively from one end of the outcrop to the other (data denoted for instance from $a$ to $z$ ).
(2) Define the orientations of cone axes by sliding triplets of data points progressively along the measurement sequence by one observation ( $a b c, b c d, c d e$, def and so on) hence producing 'running cones' (Fig. 3). The spatial sequentiality of the data set is a requirement of the method, however, the distances between measurements may be variable; data disordered with respect to the natural sequence produces results that become progressively random and more similar to the cumulative distribution of the entire data set. In other words the same data, in a randomised sequence, do not yield the same results as those obtained from the natural sequence, which reveal fold geometries not readily apparent when the entire data set is analysed together.
(3) The geometry of the conical fold axes defines the enveloping surface in which they lie thus representing an approximation to the larger-fold surface (Figs. 1 and 3). The orientations of the cone axes, if constant, define a plane that is part of a larger fold surface and may be approximated by a pole to a plane. If the surface is conical its apical angle is a measure of deviation from being planar.

Cones defined by data with similar orientations poorly constrain the geometries of lower-order folds. Therefore, at the beginning of each step of the analysis, subsequent data displaying angular discordances less than a given threshold value, i.e. sub-parallel data, were discarded (Fig. 4). The removal from the set of the data below the threshold may impact the results of the analysis. Increasing the threshold above which samples are deemed to be acceptable increases


Fig. 3. Equal-area, lower-hemisphere projections, displaying a synthetic example that illustrates how the running-cone method works. (a) Poles to folded surface (squares) with its best-fit girdle; (b) the same pole distribution (squares) treated with the running-cone method showing conical-fold axes (triangles) for the first-step of the procedure ( $a b c, b c d, c d e$ ). Only one of the four possible cones determined by three directions are shown (see Fig. 2), and the second-step conical pole (circle) to the fold axes.
the speed of the analysis but may yield results different from those obtained using smaller angular threshold values. Therefore, it is suggested that in order to minimize the loss of useful information during the analysis the criterion for the choice of the appropriate threshold be that of the smallest apical angle that produces similar orientational results towards the end of the analysis. The application of this procedure affects predominantly the data from the flatter sections of the folds, thus enhancing the high-curvature sections.

In addition to the reduction in the number of data due to the decision above, each step produces the loss of the initial and final data points of the sequence, leading to $a$ progressive decrease in the number of data, each one of them representing the 'average' of triplets of measurements.


Fig. 4. Equal-area, lower-hemisphere projection illustrating a synthetic example depicting the use of the threshold to deplete the sequence from nearly parallel fold surfaces performed at each step of the analysis in order to enhance the high-curvature sections of the folded sequence. The broken line depicts the threshold apical-angle cone, which permits us to establish that the angular divergence between the $c$ and the $d$ orientations is below the threshold and thus that $d$ is discarded to construct only the $a b c$ - and the $b c e$ cones. Only one of the four possible cones determined by three directions are shown (see Fig. 3).
(4) Cone axes that pass the threshold test may again be grouped into spatially ordered sets of three, which are plotted on a stereonet to determine poles to the conical surface. Serial steps of the method, starting from a data set of observed pole orientations, provide first a set of conical fold axes, then a set of poles to a conical surface, then again a set of fold axes and so on.

The procedure may be applied to yield useful results when the cumulative data sequence appears quite dispersed on the stereonet and the number of observations is large.

## 3. Example from the Dolomites

### 3.1. Geological outline

Here we use fold data from the Badia Valley in northeastern Italy to test the robustness of the runningcone method. The Badia Valley trends $\mathrm{N}-\mathrm{S}$, approximately normal to the main folding (Fig. 5), and incises into up to 1500 m of the lowest part of the Alpine sequence from Paleozoic metamorphic basement to the Upper Triassic dolomites, with Jurassic limestones typically covering the surrounding highlands.

About half of the way along the Badia Valley, in the vicinity of the village of Pidrò (Pederoa), a zone of folded Lower Triassic limestones is exposed and may mark the presence of the Funes Line, a major 'Störung (disturbance)' belt of regional extent (Mojsisovics, 1879; Kober, 1908; Furlani-Cornerlius, 1924). The belt trends mainly E-W but in the east consists of several NW-SE segments (Mutschlechner, 1932; Olgivie Gordon, 1934). The Pidrò folds are seen to be conical with their geometries probably reflecting the interference of multiple fold sets.

The running-cone method may reveal whether the conical shape of the folds, which were controlled by dominant $\mathrm{E}-\mathrm{W}$ vertical axial surfaces, is also influenced by interference with NW-SE structures.


Fig. 5. Simplified geological map (modified from Mutschlechner, 1932). Inset equal-area, lower-hemisphere stereoplot shows all poles to bedding (circles) together with axial surfaces (girdles) measured on the Pidrò folds exposure; data used to test the running-cone method.

### 3.2. Data and analytical procedure

The strike-and-dip data used for the present analysis were collected from S to N along an approximately $20-\mathrm{m}$ long subvertical exposure on the left bank of the Gran Ega (Gader) stream at Pidrò. Folds within the exposure have chevron geometries with upright axial surfaces that strike E-W (Fig. 6; see also stereonet inset in Fig. 5). The folds are symmetrical with metre-scale amplitudes and wavelengths. Folded strata have centimetre-to-decimetre thicknesses and typically have fractured hinges. The folded surfaces are in general easily accessible and measurements were taken at decimetre spacing where the bedding showed orientational changes.

Computations were carried out to find the cone axis orientations of triplets of adjacent data points from 150 original bedding measurements. The same procedure was applied to the cone axes determined in step one, thus
obtaining the pole orientations to the lower-order surface and further on.

Particular attention was paid to differences in cone orientations arising from the removal of data from the analysis when pairs of fold-surface orientations displayed only a small angle of divergence. In the Pidrò example the data were analysed, adopting in turn different angular distance thresholds $\left(10^{\circ}, 12^{\circ}, 14^{\circ}, \ldots\right)$ and the optimum value proved to be $16^{\circ}$ since higher values produced, after four runs, similar orientations (Fig. 7).

The results of the running-cone analysis on the Pidrò folds (Fig. 8b and c) show that they owe their conical shape to the interference of three larger fold systems unrecognisible from the 2-D riverbank outcrop. The trends revealed by the three fold axes (Fig. 8c) are likely to be also responsible for the interference pattern of the nearby Wengen Basin, clearly visible in the simplified geological map (Fig. 5, see in particular the Anisian strata). A further step in the analysis would show that the pole to the firstorder plane plunges steeply to the WNW, which is likely to correspond to the orientation of the Pidrò folds' enveloping surface.

The whole area of the Dolomites is believed to have been involved in three tectonic stages. A mesozoic $\mathrm{N}-\mathrm{S}$, normal faulting phase is reported to have been followed by a paleogene WSW-shortening phase and by a SSE-shortening in the neogene (Doglioni, 1987). In particular such tectonic phases were also recognised in an area about 10 km ESE of Pidrò (Doglioni and Siorpaes, 1990). The only fold orientation obtained from the present analysis and not previously described in the area, i.e. the NE-plunging one, may be recognised on the geological map in the Wengen Basin, NE of Pidrò (Fig. 5). The two additional fold axis orientations probably relate to the Funes Line and have already been described in the surrounding areas, i.e. the WSW direction to the west (Leonardi, 1967) and the SEplunging fold to the east (Doglioni, 1987; Doglioni and Siorpaes, 1990).

These considerations suggest that the Pidrò fold exposure is a place where the noses of folds, characterized by different wavelengths and orientations, intersect producing conical shapes and the interference pattern apparent on the geological map (Fig. 5).

## 4. Discussion and conclusions

The use of the running-cone analysis on the Pidrò example suggests that the method might be a useful tool to disclose the orientation of larger structures; the method may be used as a standard procedure in areas where outcrop scale conical folds are common. The interpretation of results may be straightforward.

It may happen that the procedure produces orientations which converge to that of the cumulative cylindrical approximation, but in principle the lowest-order poles


Fig. 6. The $\mathrm{N}-\mathrm{S}$ section of folds in Lower Triassic rocks exposed on the true left bank of the Gran Ega by the Pidrò Bridge with equal-area, lower-hemisphere stereoplots showing poles to bedding in different parts of outcrop. Data plotted in Figs. 7 and 8 were sampled from this section. Drawing from photographs looking to the west.
obtained this way would represent the successive enveloping surface orientation, while the corresponding axes would represent those of the larger enveloping folds, which may, or may not, be co-axial with their smaller counterparts.

In the analysis, the latter of any two adjacent subparallel measurements were discarded to avoid the generation of illdefined cones. Any three orientations fully define four different cones due to all the alternative combinations of the positive and negative sides of the data. The addition of a fourth orientation in the sequence generally permits the identification of the correct cone axis (Fig. 2).

Alternatively four or more data taken together may be approximated by a single cone minimising the angular
distance between its axis and the individual directions (Ramsay, 1967), thus yielding 'average' cones. The larger the number of data considered together, the higher the degree of approximation, up to obtaining a single cone representing the entire data set with local geometries becoming increasingly smoothed. It should be stressed, however, that although the consideration of more than three data together speeds up the analysis, the increasingly larger number of orientations used for the determination of single cones ought to take into consideration the higher number of alternative cones formed by the combinations of both the positive and negative sides of each direction (see Fig. 2).


Fig. 7. Equal-area, lower-hemisphere projections of poles to the lower-order surfaces derived from cone axes of the Pidrò fold train obtained by applying the running-cone method with progressively larger apical-angle thresholds to deplete the sequence from successive subparallel orientation. The different threshold cone apical angles were: (a) $0^{\circ}$; (b) $10^{\circ}$; (c) $12^{\circ}$; (d) $14^{\circ}$; (e) $16^{\circ}$; and (f) $18^{\circ}$. Note that the pole distribution becomes stable with a threshold larger than $16^{\circ}$.


Fig. 8. Equal-area, lower-hemisphere projections. Poles to bedding after four steps (a) and after six steps (b) together with the final axes after seven steps (c). All results were obtained using a threshold of $16^{\circ}$.

Similarly, excessively averaged results produced by sliding the cone data window by more than one measurement at a time may significantly degrade the fold geometries.

In conclusion it appears that the running-cone method permits the geometrical characterization of the conical folds and reveals the possible existence of larger folds, unrecognisable at outcrop scale from 2-D data. Additionally the intermediate steps of the analysis may offer information about the geometries of conical folds between outcrops and regional geological map scales.

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